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Transient Noise-Induced Optical Bistability

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Transient noise-induced optical bistability

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We show that, as a consequence of critical slowing-down and noise, the intensity probability distribution becomes double-peaked during its approach to the single-peak steady-state distribution. Even for very low noise levels, the switching time undergoes remarkable fluctuations. On average it is shorter than predicted by the deterministic theory, hence noise counteracts, in part, critical slowing-down.

The phenomenon of critical slowing down in optical bistability (o.b.), theoretically predicted by Bonifacio & Lugiato (1976), was extensively studied theoretically (Bonifacio & Meystre 1978; Benza & Lugiato 1979) and observed experimentally (Garmire *et al.* 1979; Barbarino *et al.* 1982; Grant & Kimble 1983; Mitschke *et al.* 1983). However, no analysis of the effects of noise on critical slowing-down has been made so far, even though it is quite important for the switching behaviour of the system. So we consider the simplest model that describes amplitude fluctuations in absorptive o.b.:

$$\frac{\partial P(x,\tau)}{\partial \tau} = \left\{\frac{\partial}{\partial x}\left(x-y+\frac{2Cx}{1+x^2}\right)+q\frac{\partial}{\partial x^2}\right\}P(x,\tau). \tag{1}$$

In this Fokker–Planck equation $x(y)$ is the normalized amplitude of the transmitted (incident) field, $P(x,\tau)$ is the probability distribution of the variable x at time τ, where τ is normalized to the cavity build-up time, $C = \alpha L/(2T)$ is the bistability parameter, where α is the absorption coefficient per unit length, L the length of the atomic sample and T the mirror transmissivity coefficient. The form of the diffusion term corresponds to Gaussian white noise and the diffusion coefficient q measures the noise level.

We solved (1) numerically with the conditions $P(x,0) = \delta(x)$, $C = 20$, $y = 21.04$ and several values of q. The operating value $y = 21.04$ is slightly larger than the switching-up threshold $y_{\mathrm{M}} = 21.0264$ (figure 1). The three-dimensional figure 2 shows the time evolution of $P(x,\tau)$ for $q = 0.1$. Initially the distribution is single-peaked, but soon it develops a long tail and subsequently becomes double-peaked. Finally the left peak disappears and the distribution approaches the steady-state single-peaked configuration. Hence there is an observable time interval during which the probability distribution becomes double-peaked. We call this phenomenon *transient noise-induced optical bistability*.

It is well known that the steady-state intensity probability distribution is double-peaked when y is in the bistable region $y_{\mathrm{m}} < y < y_{\mathrm{M}}$ (Bonifacio & Lugiato 1978; Schenzle & Brand 1978). However, the observation of the steady-state bimodality for $y_{\mathrm{m}} < y < y_{\mathrm{M}}$ is hard because the lifetime of the two metastable states is tremendously long. Here, instead, the observation is accessible because the phenomenon arises in the transient.

We stress that the transient bimodality, which arises exclusively from noise, is a phenomenon of general type. Nicolis and colleagues (Baras *et al.* 1983; Frankowitz & Nicolis 1984) first

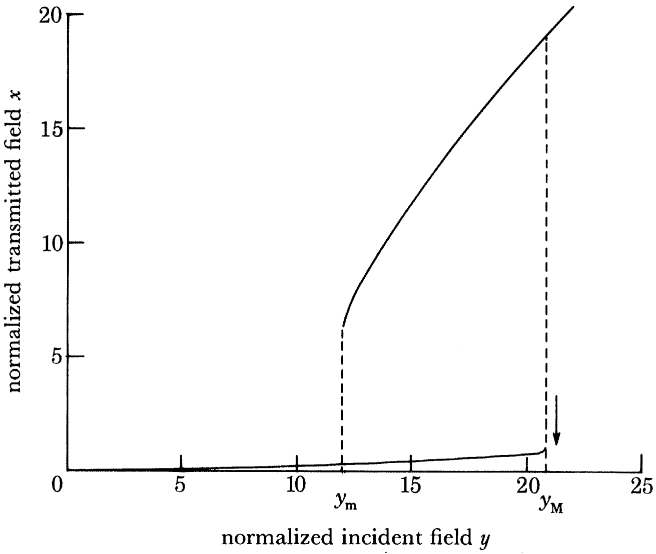

FIGURE 1. Hysteresis cycle of normalized transmitted field x as a function of normalized incident field y for $C = 20$. The arrow indicates a value of the incident field slightly larger than the switching-up threshold y_M.

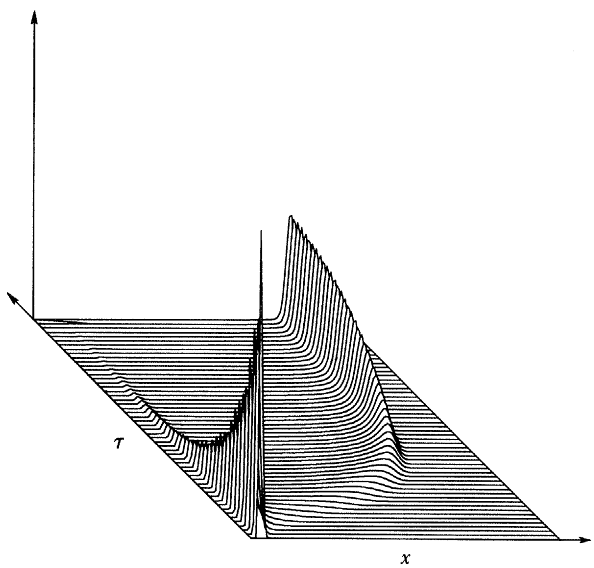

FIGURE 2. Time evolution of the probability distribution $P(x, \tau)$ for $q = 0.1$. Arbitrary units. In this, and in figures 3 and 4, $C = 20$, $y = 21.04$.

predicted it for combustion, and suggested that the same phenomenon arises whenever the solution involves a slowing-down stage followed by a rapid switching to a final single stable attractor. Hence a large variety of physical, chemical, biological, etc., systems can exhibit this phenomenon. The specific interest of o.b. in this context is that it is a promising candidate for an observation of this effect.

Another main result of our analysis is that the switching time undergoes remarkable fluctuations. As shown in figure 3, even when q is as small as 10^{-3} the switching time distribution is very broad, and the average switching time is sensibly smaller than the one predicted by the deterministic theory. This is evident from figure 4, on comparing the deterministic time

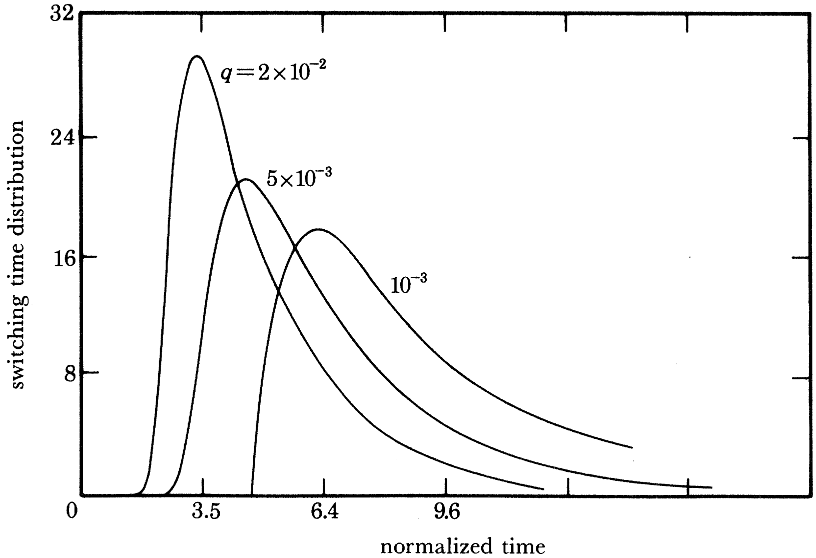

FIGURE 3. Switching time distribution for different values of q.

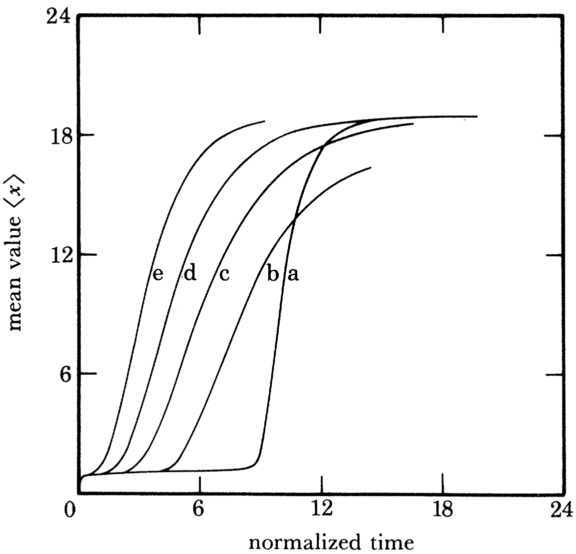

FIGURE 4. The evolution of the mean value $\langle x \rangle$ of the transmitted field when the incident field is changed stepwise from zero to the value 21.04: a, deterministic theory, $q = 0$; b, $q = 10^{-3}$; c, $q = 0.05$; d, $q = 0.02$; e, $q = 0.1$.

evolution of the mean value $\langle x \rangle (\tau)$ for different values of the noise level q. Even for $q = 10^{-3}$ the length of the horizontal critical slowing-down plateau is dramatically smaller than the one of the deterministic curve. These results suggest that the transient bistability phenomenon, as well as the presence of a broad switching time distribution, persist for $q = 10^{-4}$ and even smaller.

Our analysis shows that even very little noise produces dramatic fluctuations on the behaviour of the system, when critical slowing-down is involved. The noise effects described here seem accessible to experimental observation. Finally, since the critical slowing-down condition is so sensitive to noise, it can even be used to estimate the noise level of the system. A more detailed description of this analysis is given in Broggi & Lugiato (1984).

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